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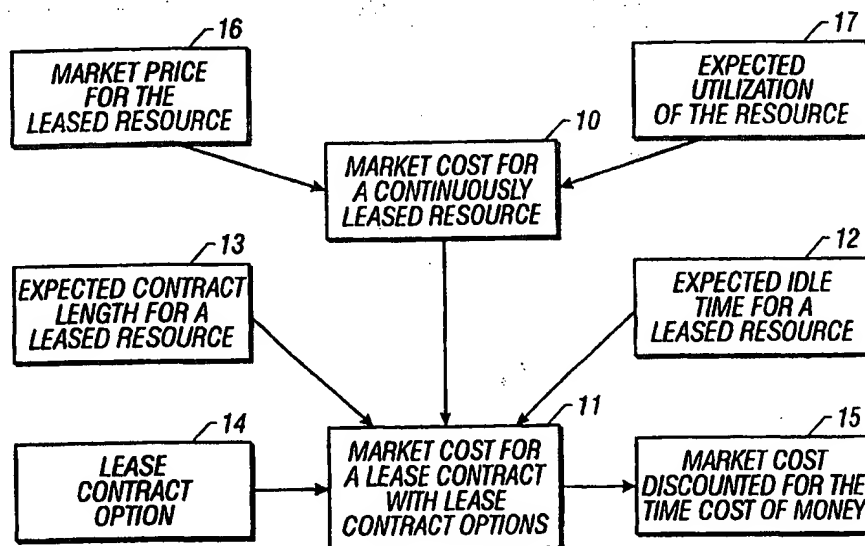
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(54) Title: EFFECT OF IDLE TIME FOR PRICING LEASE CONTRACTS AND LEASE CONTRACT OPTIONS



## (57) Abstract

A computer-implemented method for using the effect of idle time for pricing lease contract options and lease contracts with lease contract options. Leased resources include any services, equipment or real estate that may be subject to a leasing contract. The method prices a lease contract with lease contract options by determining a market cost for a continuously leased resource and calculating a market cost for the lease contract with lease contract options based upon expected leased resource idle time, expected leased resource contract length and the market cost for the continuously leased resource and the state of the market. The market cost for the lease contract may also be used as a factor to influence the price of the lease contract options.

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## EFFECT OF IDLE TIME FOR PRICING LEASE CONTRACTS AND LEASE CONTRACT OPTIONS

### BACKGROUND

5           This invention relates generally to the pricing of lease contracts for leased resources. More particularly, the invention is a computer-implemented method for using the effect of idle time for pricing lease contract options on short-term leases for leased resources.

          Repeated, short-term leases are a feature of some businesses, such as  
10   apartment leasing or services that involve capital intensive equipment performing specific functions. The terms "resource" or "leased resource" as used herein refer to any service, equipment or real estate that may be subject to a leasing contract and the term "short-term lease" means short term relative to the life of the resource. Examples of leased resources include leases for services (such as  
15   marine drilling and marine seismic services), human services (such as personnel for temporary employment), equipment (such as semi-submersible drilling rigs), buildings (such as corporate real estate leasing, retail space leasing and apartment leasing), space within buildings, manufacturing capacity (such as semiconductor manufacturing), vehicles (such as cars rented by the day), space  
20   within vehicles (such as delivery space within trucks), computer storage space (such as disc space) and communication capacity (such as bandwidth and radio spectrum). In the case of marine drilling and three-dimensional marine seismic exploration, for example, operators require services that are short-term relative to

the life of the equipment and may be repeated at different locations. These leases often include options, particularly extension, termination and assignment (or sublet) options. Extension options are important because, for example, an operator may want to continue a drilling program if favorable results are obtained

5 from an initial contract. Fixed costs involved in terminating and re-initiating exploration make the option valuable to the lease holder (operator), but expose the owner of the equipment to the possibility of idle time. The renewal price for the lease extension may be the current market rate, a fixed price set at the time of the purchase of the option or may be left open to negotiation. Termination options

10 may be used when the operator wants to be able to cancel a project for whatever reason. Assignment options give the right to sublet the subject of the lease to another party.

Systems and methods for pricing financial options exist, but the mechanisms used in pricing financial options are not applicable to leased

15 resources because they do not consider the effect of idle time. Idle time between leases relative to the market state is a critical factor in pricing options. Market state means the level of resource utilization and both the actual and expected future demand. In markets with a finite supply of resources to be leased, holding a contract option may be valuable to the lease holder to be assured that the

20 resource will be available when needed, but the lease holder does not want to pay more for a contract option than its value in the market. From the point of view of the owner of the resource, the granting of a contract option may increase the risk

that the resource will be idle between lease contracts and the price of the option should reflect that risk. Although idle time is critical to the pricing of lease contracts with contract options and the pricing of the contract options themselves, idle time has not generally been considered as a factor in pricing.

5

## SUMMARY

The present invention accurately models the effect of idle time for the pricing of contracts options and allows a price for lease contracts with contract options to be determined that reflects the effect of idle time. Idle time can act as a positive or negative modifier on extension option prices and typically as a positive  
10 modifier on termination and assignment options. A positive modifier can lead to an increased option price and a negative modifier may be used to offer more competitive contracts. The invention can model the effect of idle time in lease contracts with multiple options and combinations of different types of options. A complementary feature to idle time is equipment availability. If these two factors  
15 are neglected, the value, for example, of an option to lease equipment at the future market rate is zero. However, from the point of view of the lease holder, in markets with finite supply of equipment there may be no equipment available. For the equipment owner on the other hand, idle time associated with the granting of extension options whether or not they are exercised can make such options  
20 expensive. In an assignment option, the first operator may sublet to the second operator and so increase idle time by removing future demand. This is limited when the sublet must be wholly within the original lease. A termination option may

have a smaller notice period than the period in which operators are willing to arrange contracts and the expected idle time between contracts will increase. The present invention analyzes the effect of contract structure on both the expected idle time until the next contract and also the expected utilization from the start of  
5 the original contract until the start of the next contract.

The basis for option pricing is the idea that the total price of any contract (including options) should result in the expected rate for the contract, from when it starts to when the next contract starts, being equal to that for a market resource that is continuously leased modified by the market utilization. When discounting  
10 for the time cost of money (interest rates) is included then the idea is equality of value for the contracts. There is a clear rate for any fixed term contract. Contracts with options may put the price for the option into the rate for the duration of the contract or alternatively the option may be priced separately. Additionally, the option cost may include a penalty for not exercising the option (for example for  
15 extension options) or a discount if the option is exercised. The pricing structures are flexible based on particular business objectives.

The present invention comprises a computer implemented method for pricing a lease contract with lease contract options for a leased resource, the method comprising the steps of: determining a market cost for a continuously  
20 leased resource and calculating a market cost for the lease contract with lease contract options based upon expected leased resource idle time, expected



leased resource contract length and the market cost for the continuously leased resource.

The computer implemented method for pricing a lease contract with lease contract options for a leased resource may also comprise based on an expected rate of arrival of lease contract requests, requirements of the lease contract requests and current leasing market for the resource: determining a market cost for a continuously leased resource, determining an expected idle time and expected contract length for the leased resource, and calculating a market cost for the lease contract with lease contract options based upon the expected idle time, expected contract length, the market cost for the continuously leased resource and a probability of lease contract option exercise. The market cost for the lease contract may be used as a factor to influence the price of the lease contract with lease contract option.

The leased resource may be services, equipment, buildings, space within buildings, vehicles and space within vehicles. The leased resource may be computer storage space or communication capacity. The leased services may be human services, such as temporary personnel. The leased resource may be manufacturing capacity.

The method may further comprise modifying the market cost for the lease contract to discount for the time cost of money. The market cost for the continuously leased resource may be based upon a market price for the resource and upon an expected utilization of the resource. Besides being based

upon the expected idle time, expected contract length and the market cost for a continuously leased resource, the market cost for the lease contract with lease contract options may also include the probability that the lease contract option will be exercised and a payment schedule. The current leasing market may

5 include knowledge about existing contracts, known future contracts, and expected future contracts.

The lease contract options may be extension contract options, termination contract options and assignment contract options. The lease contract may include multiple options of the same or different types.

10 For a payment schedule, the market cost of the lease contract with contract options is used to calculate a price according to the payment schedule. The difference between the price and the market cost is used to influence the price of the lease contract options. The payment schedule may comprise: payment for the lease contract option at the start of the lease contract, payment

15 for the lease contract options together with the lease contract, and payment for the lease contract options at the time of exercise of the contract options, if the lease contract option is exercised or payment for the lease contract option at the end of the lease contract, if the lease contract option is not exercised. Payment may be contingent on lease contract option exercise and if the lease contract

20 option is exercised, payment may be contingent upon time of lease contract option exercise. For a payment schedule and specified number of payments for the lease contract with lease contract options, the market cost for the lease

contract with lease contract options may be used to determine a new market cost for the lease contract with contract options and the difference between the market cost and the new market cost may be used to modify the payments schedule and specified number of payments.

- 5           The method may further comprise calculating a market cost for the lease contract without lease contract options based upon the expected idle time, expected contract length, and market cost for a continuously leased resource and calculating the cost of the lease contract options as the difference between the market cost for the lease contract without contract options and the market
- 10   cost for the lease contract with lease contract options.

The expected idle time for the leased resource may be an estimated amount of time that the leased resource is not leased between the end of a first contract and the start of a second contract. The expected rate of arrival of lease contract requests may be modeled according to a general stochastic process.

- 15   The expected rate of arrival of lease contract requests may be expected arrivals of lease contracts. The expected rate of arrival of lease contract requests may be modeled according to a constant arrival rate the constant arrival rate may be modeled according to a Poisson process with constant rate  $\lambda$ . The expected rate of arrival of lease contract requests may be modeled according to an arrival
- 20   rate that changes with time the arrival rate that changes with may be modeled according to a non-homogeneous Poisson process.

Alternatively, the expected rate of arrival of lease contract requests may be modeled using a queuing theory model. The queuing theory model may comprise generating lease contracts and generating a list of required resources for each lease contract and assigning the lease contracts to one or more queues  
5 that hold one or more leased resources based on the list of required resources. The lease contracts may be assigned to leased resources according to a predefined priority scheme and the predefined priority scheme may be on a first come-first served basis.

The present invention includes computer executable software code stored  
10 on a computer readable medium for pricing a lease contract with lease contract options. The present invention also includes a computer-readable medium having computer-executable software code stored thereon for pricing a lease contract with lease contract options.

#### BRIEF DESCRIPTION OF THE DRAWINGS

15 These and other features, aspects and advantages of the present invention will become better understood with regard to the following description, appended claims and accompanying drawings where:

Fig. 1 is a block diagram of determining the effect of idle time on market cost for a lease contract with lease contract options.

20 Fig. 2 is a flow chart of a method of pricing a lease contract with lease contract options.

Fig. 2A is a flow chart of a method of pricing a lease contract option for a given payment schedule.

Fig. 3 is a block diagram of lease contract option types.

Fig. 4 is a block diagram of leased resource types.

5 Fig. 5 is a block diagram of expected arrival rates of lease contract requests.

Fig. 6 is a flow chart of pricing a lease contract option.

Fig. 7 is a flow chart of the queuing theory arrival rate model.

Fig. 8 shows a general simulation model based on queuing theory.

10 Fig. 9 shows an example of a general simulation model based on queuing theory.

Fig. 10 shows an example of a general simulation model based on queuing theory.

15 Fig. 11 shows an example of a general simulation model based on queuing theory.

#### DETAILED DESCRIPTION OF THE DRAWINGS

Fig. 1 is a block diagram of determining the effect of idle time on market cost for a lease contract with lease contract options. The market price for the leased resource 16 and expected utilization 17 of the resource are used to  
20 determine a market cost for a resource (with no contract options) that is assumed to be continuously leased 10. In order to determine a market cost for a lease contract with lease contract options 11, it will be assumed that the resource will

be leased for prices that result in a rate equivalent (that is, price per unit time) to the market cost for a continuously leased resource without lease contract options 10. An expected contract length for the leased resource is determined 13 and an expected idle time for the leased resource 12 is also determined. The expected

5 idle time 12 may be an estimated amount of time that the leased resource is not leased between the end of a first contract and the start of a second contract. For any lease contract option 14, the market cost for the lease contract with lease contract options 11 is calculated based upon the expected idle time 12, expected contract length 13 and the market cost for the continuously leased resource 10.

10 The market cost may then be discounted for the time cost of money 15 (interest rates). The average rate for the market resource may be calculated from any given contract rate for a fixed length contract with no options. Thus

$$\frac{s(m) \times m}{m + E[w(m)]} = c_{\infty} \times u$$

15 where  $s(m)$  is the spot (current) rate for a fixed length contract of  $m$  months,  $w(m)$  is the time a contract of length  $m$  months is idle until the next contract,  $E[\ ]$  is the usual expectation operator,  $c_{\infty}$  is the (unobserved) rate for the market resource that is continuously leased, and  $u$  is the market utilization. It is assumed that  $u$  is constant and that a difference of one resource being leased has a

20 negligible effect and so  $c_{\infty}$  is also constant. The market dynamics (stochastic process governing new contract arrivals, distributions of contract types, and assignment of contracts to resources) will determine  $E[w(m)]$  and  $u$ . If we are

given any spot rate for a fixed length contract ( $w(m)$  and  $s(m)$ ),  $c_\infty$  can be calculated and so calculate  $s(m)$  for any other fixed length contract. Given a stationary (non-time dependent) market situation, a declining rate for longer contracts will be predicted.

- 5 Contract values will depend upon subjective probabilities to some degree. The most obvious of these is the probability of option exercise. If we include discounting for the time value of money and allow market utilization  $u$  to be time dependent we obtain for the value ( $V$ ) of a fixed term contract of  $m$  months  $s(m)$

$$\begin{aligned}
 10 \quad V(s(m)) &= \int_{t=0}^{t=m} s(m) d(0,t) dt \\
 \int_{t=0}^{t=m} s(m) d(0,t) dt &= E_u \left[ \int_{t=0}^{t=m} c_\infty(u(t), t) u(t) d(0,t) dt \right. \\
 15 \quad &\quad \left. + \int_{x=0}^{x=\infty} \int_{y=0}^{y=x} c_\infty(u(m+y), m+y) u(m+y) d(0, m+y) dy P\{w(m)=x\} dx \right]
 \end{aligned}$$

where  $d(0,t)$  is the discount factor from time zero to time  $t$ ,  $P\{w(m)=x\}$  is the probability density that the idle time between contracts is  $x$  months when the first contract has length  $m$  months, and  $E_u$  is the expectation with respect to  $u$ .

- 20 This links a given or desired spot rate  $s(m)$  with the market rate for a continuously leased resource ( $c_\infty$ ). In this case,  $s(m)$  is taken as a fixed rate. Note that we have given the rate for a continuously leased resource as depending on time and market utilization. With non-zero interest rates  $c_\infty$  must change with time. We may

also expect it to depend non-linearly on utilization when utilization becomes very high (e.g. 100%, which is observed in some markets, such as semi-submersible rig leasing in the North Sea in 1997). This construction gives the basis for contract and option pricing: whatever the contract, the expected value of the contract

5 should be the same as that for the market resource over the equivalent length of time as modified by the market (expected) utilization.

An alternative way for determining the market cost for a lease contract with lease contract options 11 that is discounted for the time cost of money 15:

The value of a contract starting at time  $t$  in the future  $C(t, \Gamma, \Phi, P_\Phi, \Theta, P_\Theta, I, P_I)$  where

- $\Gamma$  is the current state of the market
- $\Phi$  is the set of parameters of the demand process
- $P_\Phi$  is the joint probability density function of the parameters of the demand process
- 15 •  $\Theta$  is the set of option exercise decisions
- $P_\Theta$  is the joint probability density function of the exercise decisions
- $I$  is the set of parameters for the interest rate model
- $P_I$  is the joint probability density function of the parameters of the interest rate model is given by

$$20 \quad C(t, \Gamma, \Phi, P_\Phi, \Theta, P_\Theta, I, P_I) = E_{t_{next}, P_\Phi, P_\Theta, P_I | \Gamma} \left[ \int_{s=t}^{s=t_{next}} d(0, s) c_\infty u ds \right]$$

where the expectation  $E[]$  is taken with respect to the random variables conditioned by the current market state  $\Gamma$  and

- $d(s_1, s_2)$  is the discount factor from time  $s_1$  to time  $s_2$
- $d(s_1, s_2)$ ,  $t_{next}$ ,  $c_\infty$  and  $u$  are functions of both  $s$  and the random variables  $\Phi$ ,
- 25  $\Theta$  and  $I$



- $c_{\infty}$  is the market rate for a continuously leased resource
- $u$  is the market utilization rate
- $t_{next}$  is a random variable giving the start time of the next contract .

Turning now to Fig. 2, a flow chart of a method of pricing a lease contract  
5 with lease contract options is shown 20. Based upon an expected rate of arrival  
of lease contract requests 21, lease contract request requirements 22 and the  
current leasing market for the leased resource 23, a market cost for a  
continuously leased resource is determined 27. The current leasing market for  
the leased resource 23 may include information about existing contracts, known  
10 future contracts and expected future contracts 24. Determining the market cost  
for a continuously leased resource 27 may include information about the  
expected utilization of the resource 25 and the current market price for the  
resource 26. The expected idle time and expected contract length for the leased  
resource is determined 28. The market cost for the lease contract with lease  
15 contract options is calculated 29. If the contract has a payment schedule,  
processing continues in Fig. 2A. If necessary, the market cost may be  
discounted for the time cost of money 31.

Fig. 2A is a flow chart of a method of pricing a lease contract option for a  
given payment schedule. After the market cost for the lease contract with lease  
20 contract options has been calculated (Fig. 2, step 29), for a given payment  
schedule 35 a price is calculated for the lease contract with contract options 40  
and difference between that calculated price and the market cost is used to  
influence the contract option price. The given payment schedule 35 may be

selected from the following: payment for the lease contract option at the start of the lease contract 36; payment for the lease contract options together with the lease contract payments 37; payment for the lease contract options at the time of exercise of the options or payment for the lease contract option at the end of the lease contract, if the option is not exercised 38; and payment contingent on lease option exercise and time of lease contract option exercise 39.

Fig. 3 is a block diagram of lease contract option types showing some of the types of lease contract options that may be include in a lease contract 42. The options include extension options 43, termination options 44 and assignment options 45. The lease contract may include multiple options of the same or different types.

Fig. 4 is a block diagram of leased resource types 46. Leased resource types include services 47, equipment 48, buildings 49, space within buildings 50, vehicles 51, space within vehicles 52, computer storage space 53, communication capacity 54 and manufacturing capacity 55.

Fig. 5 is a block diagram of expected arrival rates or expected arrivals of lease contract requests 56. Lease contract requests may be assumed to arrive according to a general stochastic process. Lease contract requests may be assumed to arrive at a constant arrival rate 57 which may be simulated by a Poisson process with constant arrival rate  $\lambda$ . Lease holders will arrange leases starting up to a months ahead, the look-ahead period. Now consider a specific contract with length  $l$  months and a notification date  $n_e$  months before the end of

the contract for an extension option of  $o$  months. There is also a minimum notification period of  $n_t$  for termination of the contract. We use  $w$  to denote the idle time from the end of a contract to the beginning of the next one for a contract with no options. Let  $w_e$  denote the idle time for a contract with an extension option and  $w_a$  and  $w_t$  denote the idle times for contracts with assignment and termination options respectively. Similarly we use  $u$  for utilization which is defined as time spent working divided by time spent working plus the idle time until the start of the next contract. Subscripts are used in the same way as for idle time so, for example,  $u_t$  is the utilization for a rig with a contract having a termination option.

10 The expected idle time from the end of a contract with no options until the next contract starts is

$$\begin{aligned}
 E[w] &= P\{\text{no requests arrive in look-ahead period}\} \\
 &\quad \times E[w|\text{no requests}] \\
 &\quad + P\{\geq 1 \text{ request arrived in look-ahead period}\} \\
 &\quad \times E[w|\geq 1 \text{ request}] \\
 &= e^{-\lambda a} (1/\lambda) + (1 - e^{-\lambda a}) \times 0 \\
 &= e^{-\lambda a} / \lambda
 \end{aligned}$$

The expected idle time for a contract with an extension option until the next contract starts is

$$\begin{aligned}
 E[w_e] &= E[w_e|\text{no exercise}] P\{\text{option not exercised}\} \\
 &\quad + E[w_e|\text{exercise}] P\{\text{option exercised}\} \\
 &= e^{-\lambda n_e} (1/\lambda) (1-p) + e^{-\lambda m} (1/\lambda) p
 \end{aligned}$$

where  $m = \min(a, n_e + o)$  and  $p$  is the probability of option exercise. In the above, it is assumed that the length of the firm part of the contract is longer than the look-ahead period. It is also assumed that the lessees will only consider firm start dates and therefore will not request a lease before the lessor has been notified as to whether the extension will be exercised.

The average utilization from the start of the contract until the start of the next contract as follows.

$$E[u] = P\{\text{no requests arrive in look-ahead period}\} \\ \times E[u|\text{no requests}] \\ + P\{\geq 1 \text{ request arrived in look-ahead period}\} \\ \times E[u|\geq 1 \text{ request}]$$

10 Now

$$E[u|\geq 1 \text{ request}] = 1$$

because there is no idle time before the start of the next contract. Also

$$E[u_e|\text{no requests, no exercise}] = E[u|\text{no requests}], \\ E[u_e|\text{no requests, no exercise}] = \int_{s=0}^{s=\infty} \lambda_e - \lambda_s \frac{l}{l+s} ds$$

15

and

$$E[u_e|\text{no requests, exercise}] = \int_{s=0}^{s=\infty} \lambda_e - \lambda_s \frac{l+o}{l+o+s} ds$$

where utilization is defined as (time working) / (time working + time idle till next contract). The integral,  $\int e^{-x}/(x+k)$  has no closed form solution and must be evaluated numerically although it is often called the exponential-integral ( $Ei$ ) function. The option price for the separate pricing contract design is then

20

calculated. This is expected to be negative (with respect to the contractor) most of the time because it is based on two fixed length contract equivalents and each of these has a part that pays for idle time at the end of the contract. Thus, if the extension option is exercised there are two payments for idle time but only one  
 5 end of contract. No discounting for the time value of money is used in the following example. The expected revenue (where we use  $v$  for the revenue, the "value", of the contract) for the market resource is

$$E[v] = (1-p) s(l, n_e) l + p s(l+o, a) (l+o)$$

on the basis that there is a probability of  $p$  (the exercise probability) of obtaining a  
 10 contract of length  $l+o$  and a probability of  $1-p$  of obtaining a contract of length  $l$ . Here  $s(a_1, a_2)$  is the spot rate for a contract of length  $a_1$  with a notification time of  $a_2$ . For a fixed length contract of length  $l$  with no options the appropriate spot rate is  $s(l, a)$  assuming that the length of the contract ( $l$ ) is longer than the period over which customers are willing to arrange new contracts ( $a$ ). The compensation for  
 15 not knowing which length contract will occur is contained in the spot rate for the shorter contract with the notification time equal to the notification time for the option ( $n_e$ ).

The spot rate used above is

20

$$\frac{s(m, n) \times m}{m + E[w(m, n)]} = c_{\infty} \times u$$

where  $w(m, n)$  is the idle time for a lease contract of length  $m$  months with a notification time, time to look for new contracts, of  $n$  months. For fixed length

contracts this is given by the length of time that customers are willing to arrange contracts ahead of the end of the contract.  $c_\infty$  is found from the market given the spot rate for some fixed length contract.

With separate pricing we have

$$5 \quad E[v] = s(l, a) l + p E[s_l(o, a)] o + v_e$$

where  $v_e$  is the value (price) of the extension option and  $p$  is the probability of option exercise as before, and  $s_l$  is the spot price at time  $l$  in the future. In this setup with no discounting and constant utilization, etc, we have  $s_l = s$ . Thus

$$10 \quad v_e = [(1-p) s(l, n_e) l + p s(l+o, a) (l+o)] - [s(l, a) l + p s(o, a) o]$$

which yields

$$\begin{aligned} v_e &= (E[w_e] - (1+p)E[w]) [c_\infty u] \\ &= ((1-p)e^{-\lambda n_e} + pe^{-\lambda a}) (1/\lambda) - (1+p)e^{-\lambda a} (1/\lambda) [c_\infty u] \\ &= (1/\lambda) ((1-p)e^{-\lambda n_e} - e^{-\lambda a}) [c_\infty u] \end{aligned}$$

15 where we have taken the case that  $o+n_e > a$ .

Tables 1, 2 and 3 give a numerical example with values that are plausible for equipment leasing such as oil rig leasing. The information in the tables includes the expected differences in idle time and average utilization for a lease contract as well as extension option price. The following assumptions are made:

20 length of the base contact is six months, the length of the extension itself is six months, the look-ahead period is six months and the notification time is one month. In the tables,  $1/\lambda$  is the average time between requests in months and  $P$

is the probability of contract option exercise. The presence of an extension option always increased the expected idle time (Table 1) before the next contract from between 0.8 months if the extension has a high probability of exercise to over 3 months if this is less. The rate of contract arrivals has a much smaller effect than

5 the probability of extension exercise for the range of parameter values shown. The values in the tables suggest that an extension option may will generally reduce the average utilization by from 13.5% at low exercise probability and high arrival rates to 1.9% at lower arrival rates and higher exercise probability (Table 2). The extension option will only increase average utilization when the probability

10 of option exercise is high, 0.8 in the table. Prices for extension options (Table 3) are positive for low probabilities of exercise and become generally negative for exercise probabilities of 0.4 and higher for average intervals between contract arrivals of 9 months and more.

Average time between requests $1/\lambda$	Probability of option exercise {extension exercise}			
	0.2	0.4	0.6	0.8
9	2.7	2.1	1.4	0.7
10	2.8	2.1	1.4	0.7
11	2.9	2.2	1.5	0.7
12	3.0	2.3	1.5	0.8

Table 1

Average time between requests $1/\lambda$	Probability of option exercise {extension exercise}			
	0.2	0.4	0.6	0.8
9	-13.5%	-8.4%	-3.3%	1.8%
10	-13.0%	-7.9%	-2.8%	2.3%
11	-12.5%	-7.4%	-2.3%	2.7%
12	-12.0%	-6.9%	-1.9%	3.1%

Table 2

Average time between requests $1/\lambda$	Probability of option exercise {extension exercise}			
	0.2	0.4	0.6	0.8
9	1.8	0.2	-1.4	-3.0
10	1.8	-0.1	-1.9	-3.7
11	1.7	-0.3	-2.4	-4.4
12	1.6	-0.7	-2.9	-5.1

Table 3

- 5           The assignment and termination options are linked in that both may be exercised at the point when the operator has no more use for a rig before the end of a contract. This is a probability that the operator at some point during the contract has no further use for the rig  $p_t$  and then the conditional distribution of the time at which this decision is reached  $f(t)$  where  $t$  runs over the contract length. It
- 10   is expected that no contract will be terminated before the first well is drilled but



- beyond this it is difficult to generalize. The expected idle time for contracts with assignment options will be increased by the loss of contracts that could have been started after the end of the current contract but are in fact used up by the assignment option. The expected idle time for a contract with a termination option
- 5 or an assignment option, for the case where the option is not exercised, is the same as if the option were not present

$$E[w_a | \text{no exercise}] = E[w_r | \text{no exercise}] = E[w]$$

$$P[\text{no exercise} | \text{option}] = 1 - p_f$$

- 10 Now when an assignment option is present and exercised

$$E[w_a | \text{exercise}] = E[w_a | \text{exercise, requests}] P\{\text{requests}\} \\ + E[w_a | \text{exercise, no requests}] P\{\text{no requests}\}$$

- 15 Note that in this case  $E[w_a | \text{exercise, requests}]$  is non-zero because a request may arrive in the look-ahead period but be used to exercise the assignment option. In this case the rig still has to wait an average of  $1/\lambda$  after the end of the original contract for the next contract request because the apparently available request is taken. Thus

$$\begin{aligned}
E[w_a | \text{exercise}] &= E[w_a | \text{exercise}, > 1 \text{ requests}] P\{ > 1 \text{ requests} \} \\
&\quad + E[w_a | \text{exercise}, 1 \text{ request}] P\{ 1 \text{ request} \} \\
&\quad + E[w_a | \text{exercise}, 0 \text{ requests}] P\{ 0 \text{ requests} \} \\
&= 0 \times P\{ > 1 \text{ requests} \} \\
&\quad + E[w_a | \text{exercise}, 1 \text{ request}] e^{-\lambda a} \lambda a \\
&\quad + e^{-\lambda a} / \lambda \\
&= ( E[w_a | \text{exercise}, 1 \text{ request, lost}] P\{ \text{lost} \} \\
&\quad + E[w_a | \text{exercise}, 1 \text{ request, not lost}] P\{ \text{not lost} \} ) e^{-\lambda a} \lambda a \\
&\quad + e^{-\lambda a} / \lambda \\
&= \frac{1}{\lambda} P\{ \text{lost} \} e^{-\lambda a} \lambda a \\
&\quad + e^{-\lambda a} / \lambda \\
&= a e^{-\lambda a} \int_{z=l_{\min}}^{z=a} f(l-z) \int_{x=0}^{x=z} H(x) dx dz \\
&\quad + e^{-\lambda a} / \lambda
\end{aligned}$$

15

The length of a contract being requested is assumed to be independent of the number of contract requests. The time left on the contract when it is available for assignment is  $z$  and  $l_{\min}$  is the length of the smallest possible contract,  $h$  is the probability density function (pdf) of contract lengths,  $f$  is the pdf described above.

20 More than one assignment in the time left in the contract to be assigned is not included here but may be. Only the case when  $z$  is less than the look ahead period has been considered, as this approximation will become increasingly inaccurate as  $z$  becomes longer. The effect will be to overestimate the idle time as a contract may be assigned before the look-ahead period and go into this period

so the probability of losing a contract arriving in the look-ahead period will be reduced. The integrals over  $x$  and  $y$  express the fact that only contracts shorter than the remaining time on the contract need be considered and that the arrival time is uniformly distributed over  $z$ . In Tables 4 through 8, the following

5 assumptions are made: length of the base contract is nine months, the length of the extension itself is six months, the look-ahead period is six months and the notification time is one month. In the tables,  $1/\lambda$  is the average time between requests in months and  $P$  is the probability of contract option exercise. Table 4 shows the increase in idle time for a contract with a termination option compared

10 to a contract without a termination option. Table 5 shows the increase in utilization for a contract with a termination option as compared to a contract without at termination option. Table 6 shows the termination option value,  $v_t$  (price) in units of  $c_{\infty}U$  months. Positive values indicate situations in which the contractor should charge a positive price for a separately priced option. Table 7 shows the increase

15 in idle time in months for a contract with an assignment option as compared to a contract without an assignment option. Even when there is a high probability of assignment exercise the effect is very small, 0.04 months in a 9 month contract. The reason for this is that few contracts are lost to the rig because they have to fit into the original contract length. We are using a lognormal for contract lengths with

20 a mean of 9 months and a standard deviation of 3 months. The assignment schedule, conditional upon exercise, is a truncated exponential. That is the

highest probability of when the assignment is exercised, given that it is exercised sometime after 1 month. This models an unfavorable initial short well.

For the termination option when it is exercised we have

$$5 \quad E[w_t | \text{exercise}] = \int_{z=n_t}^{z=l-n_t} f(z) e^{-\lambda n_t \frac{1}{\lambda}} dz$$

where  $n_t$  is the termination notice.

Expected utilization can be calculated along the same lines as for the extension option. We will use  $u$  for the utilization of a rig with a contract containing  
 10 no options and use subscripts to differentiate between utilization for different options, e.g.  $u_a$  for the utilization of a rig with a contract with an assignment option.

Therefore

$$E[u_a | \text{no exercise}] = E[u],$$

$$E[u_t | \text{no exercise}] = E[u]$$

and

$$15 \quad E[u_t | \text{exercise, requests}] = 1,$$

$$E[u_t | \text{exercise, no requests}] = \int_{z=n_t}^{z=l-n_t} f(z) \int_{s=0}^{s=\infty} \lambda e^{-\lambda s \frac{z}{z+s}} ds dz$$

The utilization for the assignment option is

$$\begin{aligned}
 E[u_a] &= E[u_a | \text{no exercise}] P\{\text{no exercise}\} \\
 &\quad + E[u_a | \text{exercise}] P\{\text{exercise}\} \\
 &= (1 - p_f) E[u] \\
 &\quad + p_f ( E[u_a | \text{exercise, no requests}] P\{\text{no requests}\} \\
 &\quad + E[u_a | \text{exercise, 1 request}] P\{\text{1 request}\} \\
 &\quad + E[u_a | \text{exercise, } > 1 \text{ requests}] P\{\text{ } > 1 \text{ request}\} ) \\
 &= (1 - p_f) E[u] \\
 &\quad + p_f ( e^{-\lambda_a} E[u | \text{no requests}] \\
 &\quad + \lambda_a e^{-\lambda_a} ( E[u | \text{no requests}] P\{\text{request lost}\} + (1 - P\{\text{request lost}\}) ) \\
 &\quad + 1 - (e^{-\lambda_a} + \lambda_a e^{-\lambda_a}) )
 \end{aligned}$$

where

$$P\{\text{request lost}\} = \int_{l_{min}}^a \int_0^s f(1-v) H(v) dv ds$$

where  $H(v)$  is the cumulative distribution function of  $h(v)$ . Higher order terms involving more than one assignment have been ignored. Table 7 and Table 8 give examples of the effects of including an assignment option in a 9 month contract when the average contract length is 9 months with a standard deviation of 3 months and the lengths are log-normally distributed. The maximum increase in idle time shown is 0.044 months or less than 1 day with a corresponding decrease in utilization of less than 0.01%.

Calculating the separate pricing option cost for the termination and assignment options and starting with the termination option we have for the value of the

contract with the termination option

$$E[v] = p_f \int_{z=n_t}^{z=l-n_t} [c_{\infty}u] (z + E[w(z, n_t)]) f(z) dz + (1 - p_f) (l + E[w(l, a)]) [c_{\infty}u].$$

- 5 Since we are charging the same rate as a fixed length contract (for separate pricing) the termination option value  $v_t$  is

$$v_t = E[v] - p_f \int_{z=n_t}^{z=l-n_t} s(l, a) z f(z) dz + (1 - p_f) (l + E[w(l, a)]) [c_{\infty}u].$$

and

10 
$$v_t = p_f [c_{\infty}u] \int_{z=n_t}^{z=l-n_t} f(z) (E[w(z, n_t)] - (z/l) E[w(l, a)]) dz$$

For the assignment option we have for the value of the contract with the option

$$E[v] = [c_{\infty}u] (l + E[w_a]).$$

- 15 Again since we are charging the same rate as a fixed length contract (for separate pricing) the assignment option value  $v_a$  is simply

$$v_a = E[v] - s(l, a) \times l$$

or equivalently

$$v_a = [c_{\infty}u] (E[w_a] - E[w])$$

20

Thus Table 7 as well as being the difference in idle times can also be read as the assignment option price in units of  $c_{\infty}u$  months.

Average time between requests $1/\lambda$	Probability of option exercise { termination exercise }			
	0.2	0.4	0.6	0.8
9	0.7	1.3	2.1	2.7
10	0.7	1.4	2.1	2.8
11	0.7	1.5	2.2	2.9
12	0.8	1.5	2.3	3.0

Table 4

Average time between requests $1/\lambda$	Probability of option exercise { termination exercise }			
	0.2	0.4	0.6	0.8
9	-8.3%	-16.6%	-25.0%	-33.3%
10	-8.2%	-16.5%	-24.7%	-33.0%
11	-8.1%	-16.3%	-24.4%	-32.6%
12	-8.0%	-16.1%	24.1%	32.1%

Table 5

Average time between requests $1/\lambda$	Probability of option exercise {termination exercise}			
	0.2	0.4	0.6	0.8
9	0.16	0.31	0.47	0.62
10	0.16	0.31	0.47	0.63
11	0.15	0.31	0.47	0.63
12	0.15	0.31	0.47	0.63

Table 6

Average time between requests $1/\lambda$	Probability of option exercise {assignment exercise}			
	0.2	0.4	0.6	0.8
9	0.009	0.018	0.028	0.037
10	0.010	0.020	0.030	0.040
11	0.011	0.021	0.032	0.042
12	0.011	0.022	0.033	0.044

Table 7



Average time between requests $1/\lambda$	Probability of option exercise { assignment exercise }			
	0.2	0.4	0.6	0.8
9	-0.0021%	-0.0042%	-0.0062%	-0.0083%
10	-0.0021%	-0.0042%	-0.0063%	-0.0084%
11	-0.0021%	-0.0042%	-0.0063%	-0.0084%
12	-0.0021%	-0.0042%	-0.0063%	-0.0084%

Table 8

Turning back to Fig. 5, lease contract requests may be assumed to arrive  
 5 56 at an arrival rate that changes with time 58 which may be simulated by a non-  
 homogeneous (or non-stationary) Poisson process (NHPP) with rate  $\lambda(t)$  rather  
 than at a constant rate  $\lambda$  as assumed above, where  $t$  stands for time. Queuing  
 effects are neglected so the system is similar to an infinite server queue. We use  
 $N(t)$  for the number of arrivals in the interval  $[0, t]$ . Now  $N(t+s)-N(t)$  is Poisson  
 10 distributed with mean  $m(t+s)-m(t)$  where  $m(t)$  is called the mean value function  
 and

$$m(t) = \int_0^t \lambda(s) ds$$

Note that  $m(t,s)$  is used for  $m(t+s)-m(t)$ .

For a contract without an option the expected idle time from the end of the contract to the beginning of the next contract is:

$$\begin{aligned}
 E[w] &= E[w|\text{no requests}] P\{\text{no requests}\} \\
 &\quad + E[w|\text{requests}] P\{\text{requests}\} \\
 &= E[w|N(l) - N(l-a) = 0] P\{N(l) - N(l-a) = 0\} \\
 &\quad + 0 \times P\{N(l) - N(l-a) > 0\} \\
 &= E[w|N(l) - N(l-a) = 0] e^{-m(l-a,a)} \\
 &= \int_{s=0}^{\infty} (1 - e^{-m(l,s)}) ds e^{-m(l-a,a)}
 \end{aligned}$$

10 For a contract with an extension option

$$\begin{aligned}
 E[w_e] &= E[w_e|\text{no exercise}] P\{\text{option not exercised}\} \\
 &\quad + E[w_e|\text{exercise}] P\{\text{option exercised}\} \\
 &= \int_{s=0}^{\infty} (1 - e^{-m(l,s)}) ds e^{-m(l-n_e, n_e)} (1-p) \\
 &\quad + \int_{s=0}^{\infty} (1 - e^{-m(l+o, s)}) ds e^{-m(l+o-n', n')} p
 \end{aligned}$$

where

$$n' = \min(o + n_e, a)$$

and  $p$  is the probability of option exercise as before.

For a contract with no options the expected utilization is :

$$\begin{aligned}
 E[u] &= E[u|\text{no requests}] P\{\text{no requests}\} + E[u|\text{requests}] P\{\text{requests}\} \\
 &= \int_{s=0}^{\infty} (1 - e^{-m(l,s)}) \frac{l}{l+s} ds e^{-m(l-a,a)} + 1(1 - e^{-m(l-a,a)})
 \end{aligned}$$

For a contract with an extension option

$$\begin{aligned}
 E[u_e] = & E[u_e | \text{no requests, no exercise}] P\{\text{no requests, no exercise}\} \\
 & + E[u_e | \text{requests, no exercise}] P\{\text{requests, no exercise}\} \\
 & + E[u_e | \text{no requests, exercise}] P\{\text{no requests, exercise}\} \\
 & + E[u_e | \text{requests, exercise}] P\{\text{requests, exercise}\}
 \end{aligned}$$

5

Assuming that exercise probability,  $p$ , is independent of contract requests

$$\begin{aligned}
 E[u_e] = & E[u_e | \text{no requests, no exercise}] P\{\text{no requests}\} (1-p) \\
 & + E[u_e | \text{requests, no exercise}] P\{\text{requests}\} (1-p) \\
 & + E[u_e | \text{no requests, exercise}] P\{\text{no requests}\} p \\
 & + E[u_e | \text{requests, exercise}] P\{\text{requests}\} p \\
 = & \int_{s=0}^{s=\infty} (1 - e^{-m(l,s)}) \frac{l}{l+s} ds e^{-m(l-n_e, n_e)} (1-p) \\
 & + 1(1 - e^{-m(l-n_e, n_e)}) (1-p) \\
 & + \int_{s=0}^{s=\infty} (1 - e^{-m(l,s)}) \frac{l+o}{l+o+s} ds e^{-m(l-n', n')} p \\
 & + 1(1 - e^{-m(l-n', n')}) p
 \end{aligned}$$

10

15

where  $n' = \min(n_e + o, a)$  as before.

Assignment and termination decisions are the results of an operator having no further use for a resource such as an oil equipment rig. As in the case of a constant contract arrival rate we will denote this probability by  $p_f$  and the distribution of times at which this decision is reached as  $f(t)$ . The idle time for a contract with either a termination option or an assignment option (where we use

20

the subscript f for both cases) is

$$E[w_f] = E[w_f | \text{no exercise}] P\{\text{no exercise}\} \\ + E[w_f | \text{exercise}] P\{\text{exercise}\}$$

Now

5

$$P\{\text{exercise}\} = p_f \\ P\{\text{no exercise}\} = 1 - p_f \\ E[w_f | \text{no exercise}] = E[w]$$

In the termination option case

$$E[w_f | \text{exercise}] = E[w_f | \text{exercise, no requests}] P\{\text{no requests}\} \\ + E[w_f | \text{exercise, requests}] P\{\text{requests}\}$$

but

10

$$E[w_f | \text{exercise, requests}] = 0$$

Note that  $P\{\text{no requests}\}$  will be a function of when the termination option is exercised as will  $E[w_f | \text{exercise, no requests}]$ . Thus

$$E[w_f | \text{exercise}] = \int_{z=n_i}^{z=l-n_i} f(z) \int_{s=0}^{s=\infty} 1 - e^{-m(z,s)} ds e^{-m(z-n_i, n_i)} dz$$

15

Now for the case of the assignment option

$$E[w_a | \text{exercise}] = E[w_a | \text{exercise, no requests}] P\{\text{no requests}\} \\ + E[w_a | \text{exercise, 1 request}] P\{\text{1 request}\} \\ + E[w_a | \text{exercise, > 1 request}] P\{\text{> 1 request}\}$$

20 and

$$E[w_a | \text{exercise, > 1 request}] = 0$$

So

$$\begin{aligned}
 E[w_a | \text{exercise}] &= E[w_a | \text{exercise, no requests}] P\{\text{no requests}\} \\
 &\quad + E[w_a | \text{exercise, 1 request}] P\{1 \text{ request}\} \\
 &\quad + E[w_a | \text{exercise, } > 1 \text{ request}] P\{> 1 \text{ request}\} \\
 &= \int_{z=0}^z f(l-z) E[w_a | \text{exercise, no requests in } z] P\{\text{no requests in } z\} \\
 &\quad + E[w_a | \text{exercise, 1 request in } z] P\{1 \text{ request in } z\} \\
 &\quad + E[w_a | \text{exercise, } > 1 \text{ requests in } z] P\{> 1 \text{ requests in } z\} dz
 \end{aligned}$$

5

Now we have

$$P\{\text{no requests in } z\} = e^{-m(l-z, z)}$$

$$P\{1 \text{ request in } z\} = e^{-m(l-z, z)} m(l-z, z)$$

$$P\{> 1 \text{ requests in } z\} = 1 - e^{-m(l-z, z)} (1 + m(l-z, z))$$

10 And also

$$E[w_a | \text{exercise, no requests in } z] = \int_{s=0}^{s=\infty} 1 - e^{-m(l, s)} ds$$

$$E[w_a | \text{exercise, 1 request in } z] = \frac{1}{m(l-z, z)} \int_{x=l-z}^x \lambda(x) \int_{v=0}^{v=l-x} h(y) dy dx$$

$$\times \int_{s=0}^{s=\infty} 1 - e^{-m(l, s)} ds$$

15

$$E[w_a | \text{exercise, } > 1 \text{ requests in } z] = 0$$

The expected utilization time for a contract with either a termination option or an assignment option (where we use the subscript f for both cases) is

$$\begin{aligned}
 E[u_f] &= E[u_f | \text{no exercise}] P\{\text{no exercise}\} \\
 &\quad + E[u_f | \text{exercise}] P\{\text{exercise}\}
 \end{aligned}$$

20

Where, analogous to the situation for idle time,

$$P\{\text{exercise}\} = p_f$$

$$P\{\text{no exercise}\} = 1 - p_f$$

$$E[u_f | \text{no exercise}] = E[u]$$

For the termination option

$$5 \quad E[u_t | \text{exercise}] = \int_{z=n_t}^{z=l-n_t} f(z) (E[u_t | \text{no requests in } n_t] P\{\text{no requests in } n_t\} + E[u_t | \text{requests in } n_t] P\{\text{requests in } n_t\}) dz$$

and

$$10 \quad \begin{aligned} P\{\text{no requests in } n_t\} &= e^{-m(z-n_t, n_t)} \\ P\{\text{requests in } n_t\} &= 1 - e^{-m(z-n_t, n_t)} \\ E[u_t | \text{requests in } n_t] &= 1 \\ E[u_t | \text{no requests in } n_t] &= \int_{s=0}^{s=\infty} (1 - e^{-m(z,s)}) \frac{z}{z+s} ds \end{aligned}$$

For the assignment option we have

$$15 \quad \begin{aligned} E[u_a | \text{exercise}] &= \int_{z=0}^{z=a} f(l-z) (E[u_a | \text{no requests in } z] P\{\text{no requests in } z\} \\ &\quad + E[u_a | 1 \text{ request in } z] P\{1 \text{ request in } z\} \\ &\quad + E[u_a | > 1 \text{ request in } z] P\{> 1 \text{ request in } z\}) dz \end{aligned}$$

20

and

$$P\{\text{no requests in } z\} = e^{-m(l-z, z)}$$

$$P\{1 \text{ request in } z\} = e^{-m(l-z, z)} m(l-z, z)$$

$$P\{> 1 \text{ request in } z\} = 1 - e^{-m(l-z, z)} (1 + m(l-z, z))$$

5

$$E[u_t | \text{no requests in } z] = \int_{s=0}^{s=\infty} (1 - e^{-m(l, s)}) \frac{l}{l+s} ds$$

$$E[u_t | 1 \text{ request in } z] = \frac{1}{m(l-z, z)} \int_{x=l-z}^{x=l} \lambda(x) \int_{y=0}^{y=l-x} h(y) dy dx$$

$$\times \int_{s=0}^{s=\infty} (1 - e^{-m(l, s)}) \frac{l}{l+s} ds$$

$$+ \frac{1}{m(l-z, z)} \int_{x=l-z}^{x=l} \lambda(x) \int_{y=l-x}^{y=\infty} h(y) dy dx$$

$$\times 1$$

10

$$E[u_t | > 1 \text{ request in } z] = 1$$

Turning back to Fig. 5, lease contract requests may also be assumed to

15 arrive 56 at an arrival rate that changes that is modeled based on queuing theory 58. The queuing theory simulation model is shown in Fig. 8.

Fig. 6 is a flow chart of pricing a lease contract option 60. Based upon an expected rate of arrival of lease contract requests 61, lease contract request requirements 62 and the current leasing market for the leased resource 63, a market cost for a continuously leased resource is determined 64. The expected idle time and expected contract length for the leased resource is determined 65. The market cost for the lease contract without contract options is calculated 66.

The market cost for the lease contract with lease contract options is calculated 67. The market cost for the lease contract options is calculated as the difference between the market cost without the lease contract option and the market cost with the lease contract option 68. If necessary the market cost may be 5 discounted for the time cost of money 69.

Fig. 7 is a flow chart of the queuing theory arrival rate model 70. A list of lease contracts is generated 71. A list of required resources is generated 72. Lease contracts are assigned to one or more queues that hold the leased resources based on the list of required resources 73. The assignments of the lease 10 contract 73 may be accomplished on the basis of a predefined priority scheme 74 or may be assigned on a first come-first served basis 75.

Fig. 8 shows a general simulation model based on queuing theory. The market resembles a multi-server queue with contracts queuing for resource time or services. In this section job and lease contract are equivalent. In the 15 simulation there is a demand process 80 that generates lease contracts (or jobs) which then go to a holding area (Job List) 81. The demand process 80 may be dependent on the state of the holding area (Job List) 81 and the queues 83-86 and resources 87-90 and time or only a subset of this information. Resources may be of the discrete or continuous type and may be single or multiple 20 resources 87-90. Jobs, including the job or contract descriptions, are created by the demand process 80 and then move to the Job List 81 which represents jobs in the tender or negotiation process. The job descriptions include the resources



required to fulfill the job. The assignment process 82 represents the tender/negotiation process and assigns jobs from the holding area (Job List) 81 to queues 83-86 for resources 87-90. Jobs may be cancelled by assignment to the Cancel box 91. In this general situation there may be only one resource in the market or there may be many similar (but not identical) resources. Jobs may be able to be fulfilled by any resource or only by some. The resource used to fulfill the job or contract may be one of several forms depending on the specific application. A job or lease contract may contain only a single option or may contain multiple options of different types. For example, a lease contract may be for 1 year with three extension options each for six months. There may be an assignment option on the base contract of 1 year and on each of the assignment options and a cancellation option on the base contract. Together with these options the joint probability density (or distribution) of their (time dependent) exercise must be given. Several different densities may be given to obtain alternative option pricing.

Fig. 9 shows an example of a general simulation model based on queuing theory. In Fig. 9, the resource (Fig. 8, 87-90) in the queue 92 is a single server 93.

Fig. 10 shows an example of a general simulation model based on queuing theory. In Fig. 10, the resource (Fig. 8, 87-90) in the queue 94 is a collection of servers 95-98. Each server 95-98 may service one job at a time. Jobs in the

queue 94 may be serviced according to a first come first served policy or another selected policy.

Fig. 11 shows an example of a general simulation model based on queuing theory. In Fig. 11, the resource (Fig. 8, 87-90) is a stock 100 of resources, such as office space or reusable equipment. The solid arrowhead represent the flow of stock. Jobs in the queue 99 only enter the stock box 100 if they require less stock than is available. The total stock may be fixed or may be some function of time that may increase and decrease. Jobs and the stock that they are using wait in the service box 101 until the job service time is elapsed. Note that several jobs may be in the service box 101 at the same time. After a job is finished (i.e. its service time has elapsed), the stock 100 reenters the stock box 100 and is available to further jobs. There may be a delay after the job finishes before the stock reenters the stock box which may be either a fixed amount or some amount determined by random variable.

Example applications include

- Semi-submersible rig leases: use single server for each resource with each server representing a single rig.
- Apartment rentals: use multiple servers for each resource where each resource represents similar apartments (with one server for each apartment) within the same apartment complex.
- Retail office space: use stock and service for each resource where each resource represents physically continuous or adjacent space in the same

building and different resources represent different buildings or different categories of office space (e.g. wired for networked computers or not).

What is claimed is:

1. A computer implemented method for pricing a lease contract with lease contract options for a leased resource, the method comprising the steps of:
  - a. determining a market cost for a continuously leased resource; and
  - 5 b. calculating a market cost for the lease contract with lease contract options based upon expected leased resource idle time, expected leased resource contract length and the market cost for the continuously leased resource.
2. A computer implemented method for pricing a lease contract with lease contract options for a leased resource, the method comprising the steps of:
  - a. based on an expected rate of arrival of lease contract requests, requirements of the lease contract requests and current leasing market for the resource:
    - i. determining a market cost for a continuously leased resource;
    - 15 ii. determining an expected idle time and expected contract length for the leased resource; and
  - b. calculating a market cost for the lease contract with lease contract options based upon the expected idle time, expected contract length, the market cost for the continuously leased resource and a probability of lease contract option exercise.
- 20 3. The method according to claim 1, further comprising modifying the market cost for the lease contract to discount for the time cost of money.

4. The method according to claim 1, wherein the market cost for the continuously leased resource is based upon a market price for the resource.
5. The method according to claim 1, wherein the market cost for the continuously leased resource is based upon an expected utilization of the  
5 resource.
6. The method according to claim 2, wherein the current leasing market comprises:
  - a. existing contracts;
  - b. known future contracts; and
  - 10 c. expected future contracts.
7. The method according to claim 2, wherein the lease contract options are selected from the group consisting of extension contract options, termination contract options and assignment contract options.
8. The method according to claim 2, further comprising using the market cost  
15 for the lease contract as a factor to influence the price of the lease contract with lease contract option.
9. The method according to claim 2, further comprising calculating a market cost for the lease contract with lease contract options based upon the expected idle time, expected contract length, the market cost for a  
20 continuously leased resource, the probability of lease contract option exercise, and a payment schedule.

10. The method according to claim 2, further comprising:

- a. for a payment schedule, using the market cost of the lease contract with contract options to calculate a price according to the payment schedule; and
- 5 b. using the difference between the price and the market cost to influence the price of the lease contract options.

11. The method according to claim 2, further comprising:

- a. for a payment schedule and specified number of payments for the lease contract with lease contract options, using the market cost for the lease contract with lease contract options to determine a new market cost for the lease contract with contract options; and
- 10 b. using the difference between the market cost and the new market cost to modify the payments schedule and specified number of payments.

12. The method according to claim 2, further comprising:

- 15 a. calculating a market cost for the lease contract without lease contract options comprising the expected idle time, expected contract length, and market cost for a continuously leased resource; and
- b. calculating the cost of the lease contract options as the difference between the market cost for the lease contract without contract options and the market cost for the lease contract with lease contract options.
- 20

13. A computer implemented method for pricing a lease contract and lease contract options for a leased resource, the method comprising the steps of:

- a. based on an expected rate of arrival of lease contract requests, specifications of the lease contract requests and current leasing market for the resource:
- i. determining a market cost for a continuously leased resource;
  - 5 ii. determining an expected idle time and expected contract length for the leased resource;
  - iii. calculating a market cost for the lease contract without lease contract options comprising the expected idle time, expected contract length and the market cost for the continuously leased resource;
  - 10 iv. calculating a market cost for the lease contract with lease contract options comprising the expected idle time, expected contract length, the market cost for the continuously leased resource and probability of lease contract option exercise; and
  - v. calculating the cost of the lease contract options as the difference
  - 15 between the market cost for the lease contract without contract options and the market cost for the lease contract with contract options.

14. A method according to claim 2, wherein the leased resource may be selected from the group consisting of leased services, equipment, buildings, space

20 within buildings, vehicles and space within vehicles.

15. A method according to claim 2, wherein the leased resource may be computer storage space.

16. A method according to claim 2, wherein the leased resource may be communication capacity.
17. The method according to claim 2, wherein the expected rate of arrival of lease contract requests is modeled according to a constant arrival rate
- 5 Poisson process model.
18. The method according to claim 2, wherein the expected rate of arrival of lease contract requests is modeled according to an arrival rate that changes with time.
19. The method according to claim 2, wherein the expected rate of arrival of
- 10 lease contract requests is modeled using a queuing theory model.
20. Computer executable software code stored on a computer readable medium, the code for pricing a lease contract with lease contract options comprising:
- a. based on an expected rate of arrival of lease contract requests, requirements of the lease contract requests and current leasing market
- 15 for the resource:
- i. code for determining a market cost for a continuously leased resource;
- ii. code for determining an expected idle time and expected contract length for the leased resource; and
- 20 b. code for calculating a market cost for the lease contract with lease contract options based upon the expected idle time, expected contract



length, the market cost for the continuously leased resource and a probability of lease contract option exercise.

21. A computer-readable medium having computer-executable software code stored thereon, the code for pricing a lease contract with lease contract

5 options comprising:

a. based on an expected rate of arrival of lease contract requests, requirements of the lease contract requests and current leasing market for the resource:

10 i. code for determining a market cost for a continuously leased resource;

ii. code for determining an expected idle time and expected contract length for the leased resource; and

15 b. code for calculating a market cost for the lease contract with lease contract options based upon the expected idle time, expected contract length, the market cost for the continuously leased resource and a probability of lease contract option exercise.

22. The method according to claim 23, wherein the leased services are human services.

23. The method according to claim 2, wherein the leased resource is  
20 manufacturing capacity.

1/7

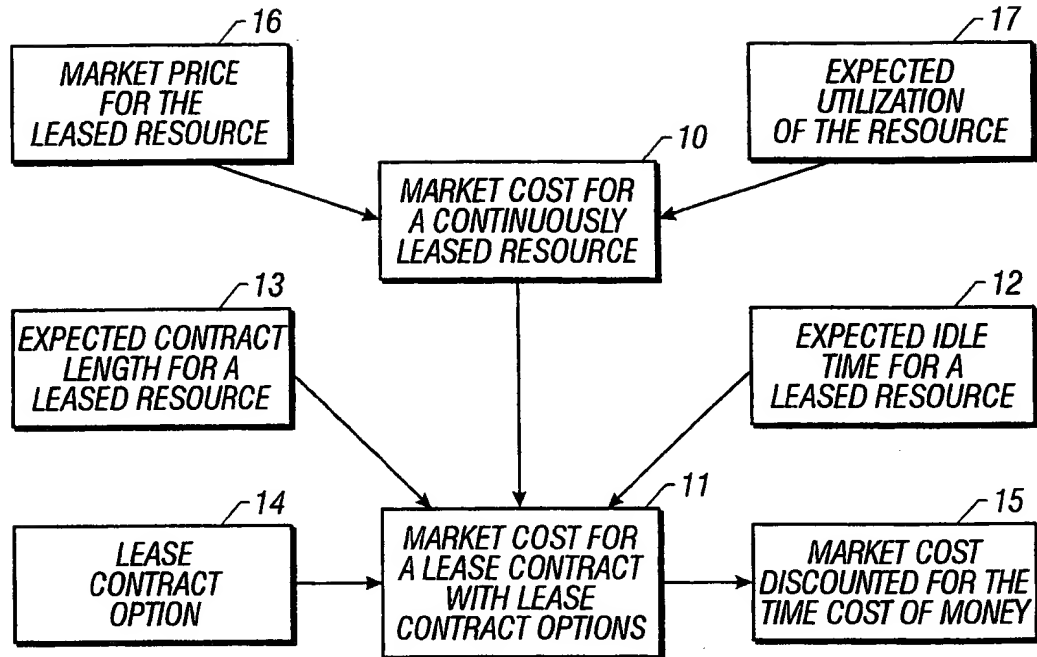


FIG. 1

2/7

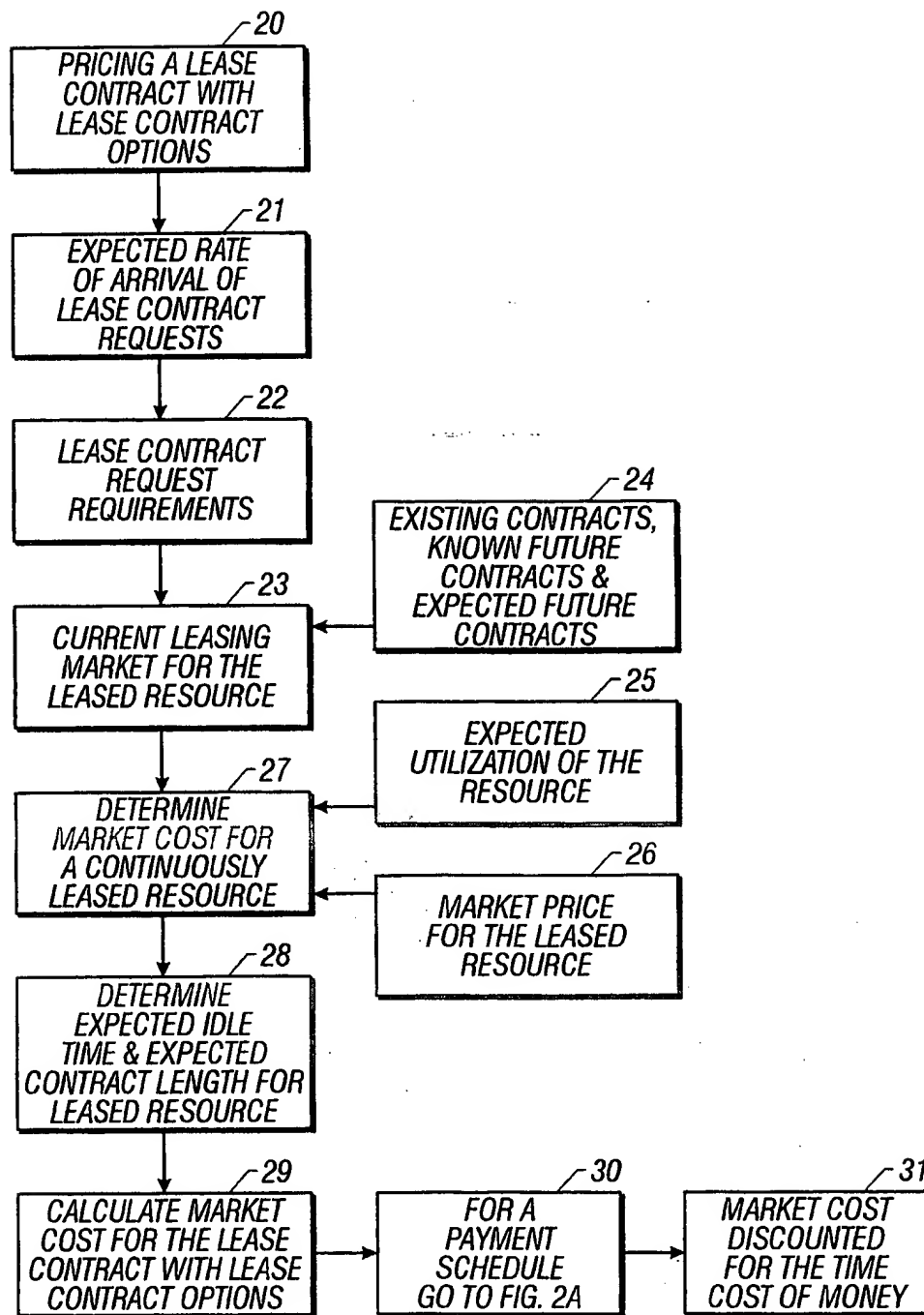


FIG. 2

3/7

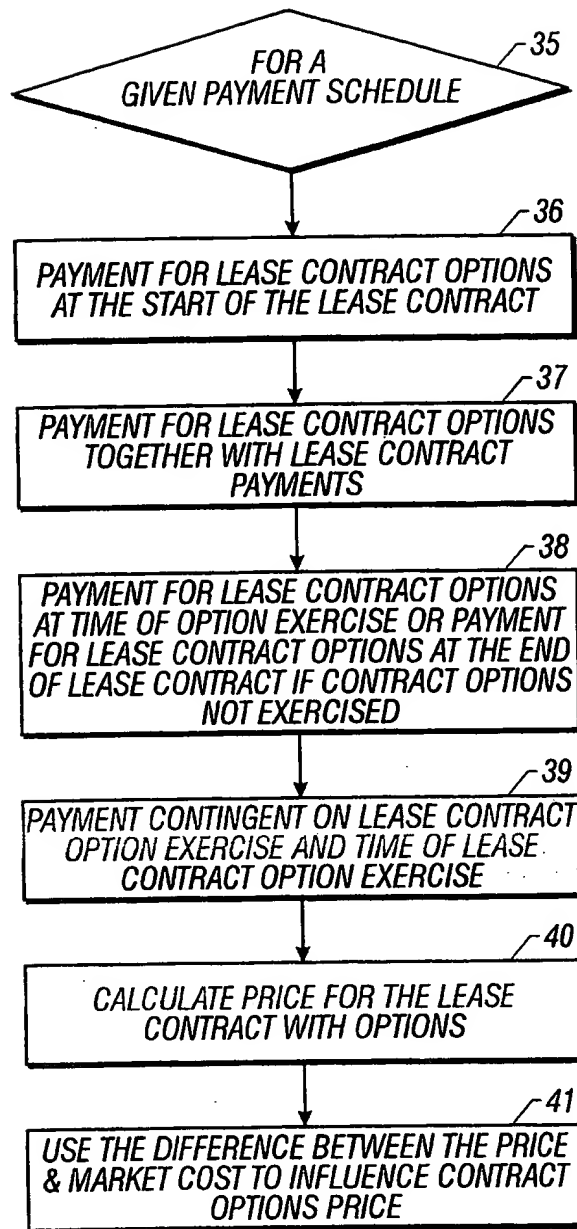


FIG. 2A

4/7

LEASE CONTRACT OPTION TYPES	42
• EXTENSION	43
• TERMINATION	44
• ASSIGNMENT	45

FIG. 3

LEASED RESOURCES	46
• SERVICES	47
• EQUIPMENT	48
• BUILDINGS	49
• SPACE WITHIN BUILDINGS	50
• VEHICLES	51
• SPACE WITHIN VEHICLES	52
• COMPUTER STORAGE SPACE	53
• COMMUNICATION CAPACITY	54
• MANUFACTURING CAPACITY	55

FIG. 4

EXPECTED RATE OF ARRIVAL OF LEASE CONTRACT REQUESTS (GENERAL STOCHASTIC PROCESS)	56
• CONSTANT ARRIVAL RATE -POISSON PROCESS	57
• ARRIVAL RATE CHANGES WITH TIME -NON-HOMOGENEOUS POISSON PROCESS	58
• ARRIVAL RATE MODELED ON QUEUING THEORY	59

FIG. 5

5/7

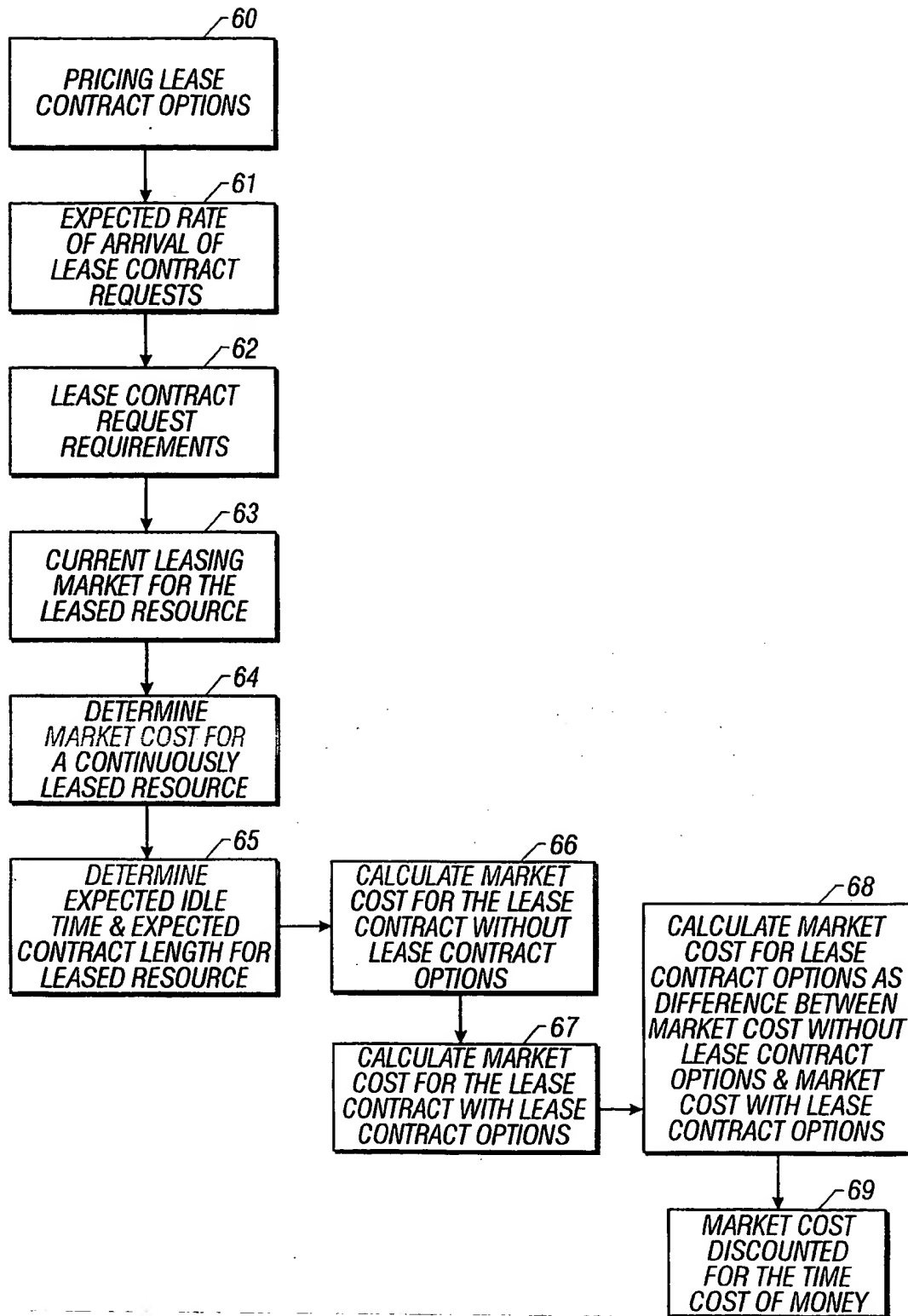


FIG. 6

6/7

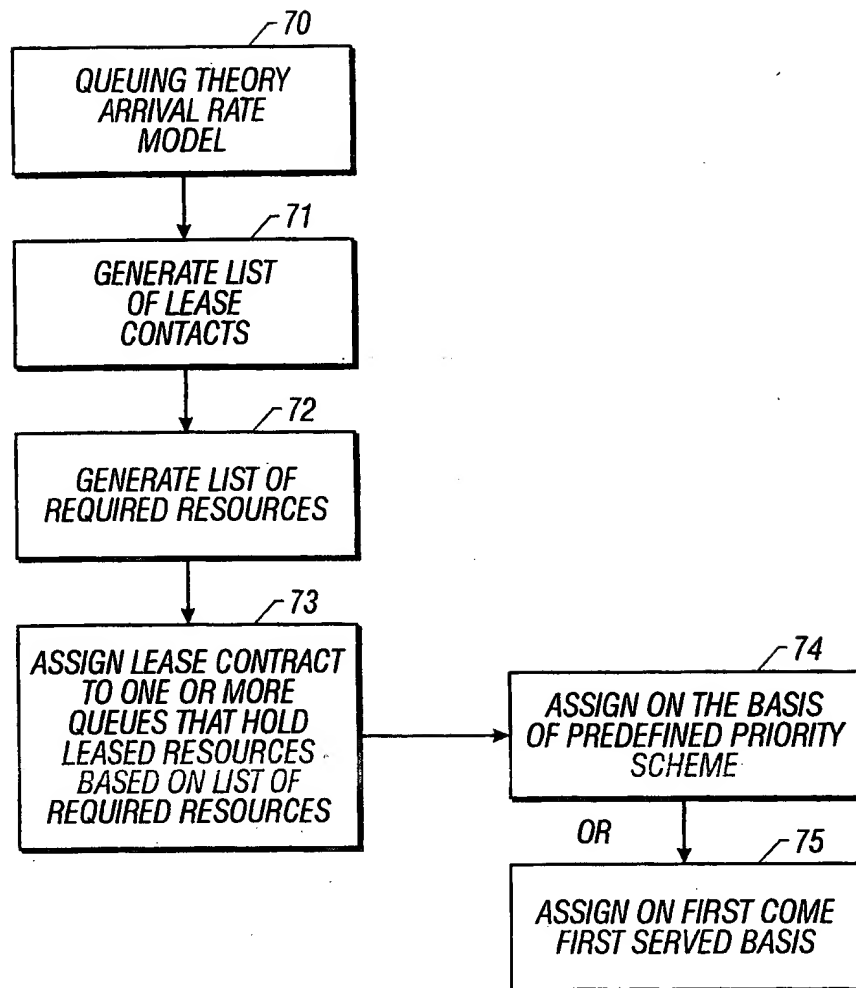


FIG. 7

7/7

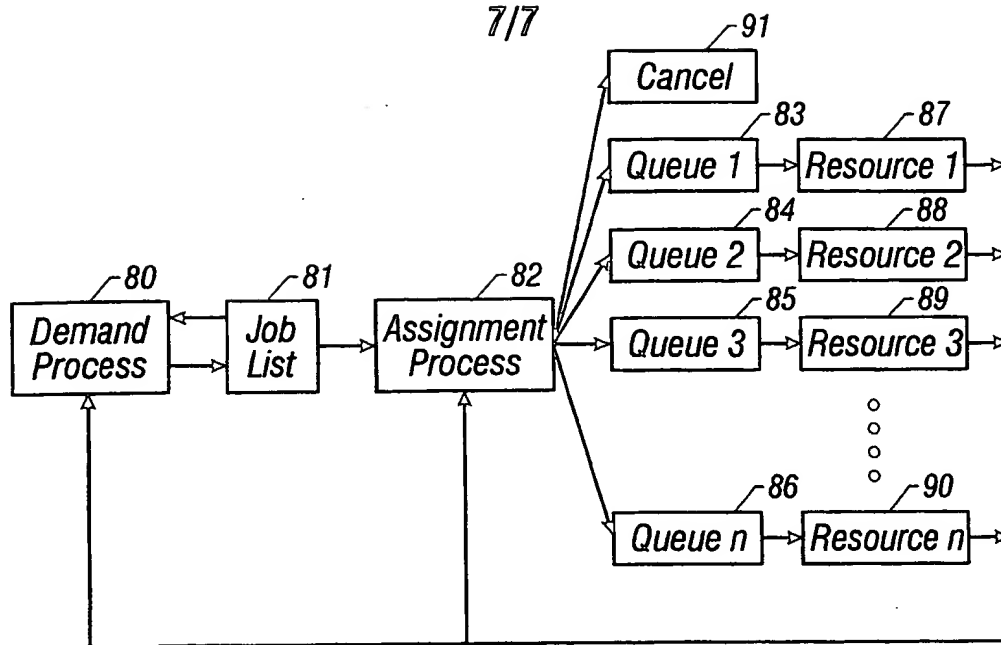


FIG. 8

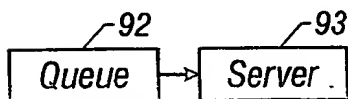


FIG. 9

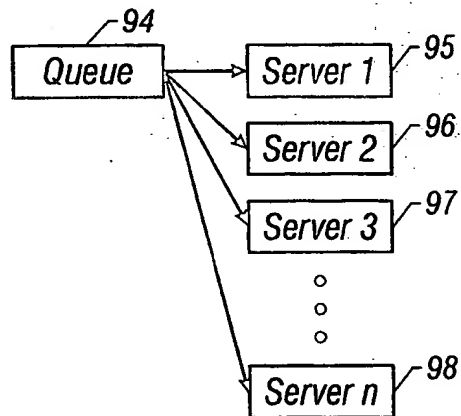


FIG. 10

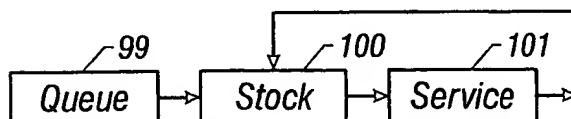


FIG. 11



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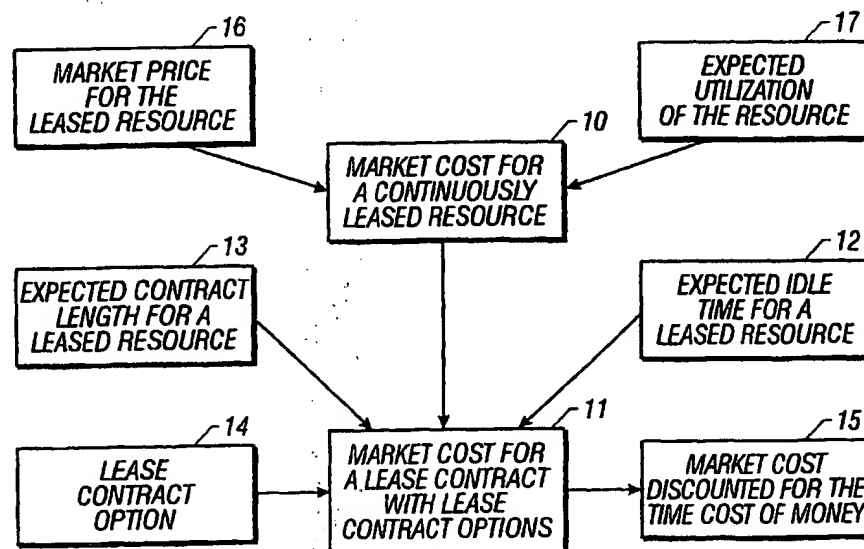
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- (72) Inventors; and
- (75) Inventors/Applicants (for US only): KENYON, Christopher, M. [GB/US]; 2008 Dayflower Trace, Cedar Park, TX 78613 (US). TOMPAIDIS, Stathis [GR/US]; 7610 Cameron Rd. #2116, Austin, TX 78752 (US).
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[Continued on next page]

(54) Title: EFFECT OF IDLE TIME FOR PRICING LEASE CONTRACTS AND LEASE CONTRACT OPTIONS



(57) Abstract: A computer-implemented method for using the effect of idle time for pricing lease contract options and lease contracts with lease contract options. Leased resources include any services, equipment or real estate that may be subject to a leasing contract. The method prices a lease contract with lease contract options by determining a market cost for a continuously leased resource and calculating a market cost for the lease contract with lease contract options based upon expected leased resource idle time, expected leased resource contract length and the market cost for the continuously leased resource and the state of the market. The market cost for the lease contract may also be used as a factor to influence the price of the lease contract options.

WO 00/43937 A3



*For two-letter codes and other abbreviations, refer to the "Guidance Notes on Codes and Abbreviations" appearing at the beginning of each regular issue of the PCT Gazette.*

# INTERNATIONAL SEARCH REPORT

International Application No  
PCT/US 00/00612

## A. CLASSIFICATION OF SUBJECT MATTER

IPC 7 G06F17/60

According to International Patent Classification (IPC) or to both national classification and IPC

## B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)  
IPC 7 G06F

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data base consulted during the international search (name of data base and, where practical, search terms used)

EPO-Internal, COMPENDEX, INSPEC, IBM-TDB, WPI Data, PAJ

## C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category *	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
P,X	KENYON C M ET AL: "Real options in leasing semi-submersible rigs in the North Sea" PROCEEDINGS OF THE 1999 IEEE/IAE CONFERENCE ON COMPUTATIONAL INTELLIGENCE FOR FINANCIAL ENGINEERING (CIFER), 28 - 30 March 1999, pages 218-239, XP002151617 NEW YORK, NY, USA page 218, line 1 -page 239, line 16 --- -/-	20,21

☒ Further documents are listed in the continuation of box C.

☒ Patent family members are listed in annex.

### \* Special categories of cited documents:

- \*A\* document defining the general state of the art which is not considered to be of particular relevance
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European Patent Office, P.B. 5818 Patentlaan 2  
NL - 2280 HV Rijswijk  
Tel. (+31-70) 340-2040, Tx. 31 651 epo nl.  
Fax: (+31-70) 340-3016

Authorized officer

Pedersen, N

# INTERNATIONAL SEARCH REPORT

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PCT/US 00/00612

## C.(Continuation) DOCUMENTS CONSIDERED TO BE RELEVANT

Category *	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
X	<p>TRIGEORGIS LENOS: "Evaluating leases with complex operating options"</p> <p>EUR J OPER RES;EUROPEAN JOURNAL OF OPERATIONAL RESEARCH JUN 7 1996 ELSEVIER SCIENCE B.V., AMSTERDAM, NETHERLANDS, vol. 91, no. 2, 7 June 1996 (1996-06-07), pages 315-329, XP000957710</p> <p>page 315, column 1, line 1 -page 328, column 2, line 13</p> <p style="text-align: center;">---</p>	20,21
A	<p>WO 97 03408 A (SHEPHERD IAN KENNETH)</p> <p>30 January 1997 (1997-01-30)</p> <p>abstract</p> <p style="text-align: center;">-----</p>	20,21

# INTERNATIONAL SEARCH REPORT

International application No.  
PCT/US 00/00612

## Box I Observations where certain claims were found unsearchable (Continuation of item 1 of first sheet)

This International Search Report has not been established in respect of certain claims under Article 17(2)(a) for the following reasons:

1. ☒ Claims Nos.: 1-19, 22-23  
because they relate to subject matter not required to be searched by this Authority, namely:  
The subject-matter of claims 1-19, 22-23 falls under the provisions of Rule 39.1(iii) PCT.
2. ☐ Claims Nos.:  
because they relate to parts of the International Application that do not comply with the prescribed requirements to such an extent that no meaningful International Search can be carried out, specifically:
3. ☐ Claims Nos.:  
because they are dependent claims and are not drafted in accordance with the second and third sentences of Rule 6.4(a).

## Box II Observations where unity of invention is lacking (Continuation of item 2 of first sheet)

This International Searching Authority found multiple inventions in this international application, as follows:

1. ☐ As all required additional search fees were timely paid by the applicant, this International Search Report covers all searchable claims.
2. ☐ As all searchable claims could be searched without effort justifying an additional fee, this Authority did not invite payment of any additional fee.
3. ☐ As only some of the required additional search fees were timely paid by the applicant, this International Search Report covers only those claims for which fees were paid, specifically claims Nos.:
4. ☐ No required additional search fees were timely paid by the applicant. Consequently, this International Search Report is restricted to the invention first mentioned in the claims; it is covered by claims Nos.:

### Remark on Protest

- ☐ The additional search fees were accompanied by the applicant's protest.
- ☐ No protest accompanied the payment of additional search fees.

# INTERNATIONAL SEARCH REPORT

Information on patent family members

International Application No

PCT/US 00/00612

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